ADDITIONAL MATHEMATICS

Paper 0606/12 Paper 12

Key messages

Candidates should be reminded of the importance of reading the rubric on the examination paper. Accuracy is of vital importance with final answers to be given correct to three significant figures unless otherwise stated in the question. This includes angles which are given in terms of radians.

General comments

Most candidates appeared to have sufficient time to complete the paper fully, with very few candidates missing out questions either through lack of either time or ability. For those candidates who required extra space to complete their work, most indicated on their exam paper in the appropriate space that the question was continued elsewhere. This was much appreciated as it enabled the marking procedure to be carried out smoothly. There were some excellent scripts and many candidates showed a good understanding of the syllabus aims and objectives.

Comments on specific questions

Question 1

- (a) Most candidates obtained full marks for this part of the question, with only the occasional slip being made. In **part (i)** some candidates failed to gain the mark if they used the symbol for the empty set, \emptyset , rather than the number 0 which was expected.
- (b) Very few incorrect Venn diagrams were produced. On the incorrect diagrams, most candidates were able to deal with the fact that $Y \cap Z = \emptyset$ but had problems when dealing with $X \cap Y = Y$ and $X \cap Z = Z$, not realising that Y and Z were both contained within X.

Answer: (a)(i) 0 (ii) 10

Question 2

(i) Most candidates made a reasonable attempt at a sketch with many completely correct sketches seen. Some candidates, however, did not correctly sketch the curve at the points $(0^{\circ},4)$ and $(360^{\circ},4)$. At these points it was expected that the gradient of the curve would be approximately

zero and not demonstrably negative at $(0^\circ, 4)$ and positive at $(360^\circ, 4)$.

(ii) Most candidates were able to write down the coordinates of the minimum point correctly, but there were some cases where the coordinates were transposed.

Answer: (ii) (90°,-2)



Question 3

Candidates showed a good understanding of the application of the binomial expansion, thus very few incorrect solutions were seen. Any errors made were usually due to the incorrect evaluation of the third term in the expansion i.e. $10 \times \frac{1}{4} \times a^3$ rather than $10 \times \frac{1}{16} \times a^3$.

Answer: a = 2, b = 20, c = 5

Question 4

- (a) Very few incorrect inverse matrices were seen and, with the exception of a few candidates, most used the matrix \mathbf{A}^{-1} correctly in order to find the matrix \mathbf{M} . Some candidates made use of simultaneous equations to obtain the matrix \mathbf{M} . Candidates need to be aware of the importance of using the method specified in questions such as this. On this occasion, candidates were required to use \mathbf{A}^{-1} .
- (b) Most candidates used the determinant of each matrix correctly in order to achieve full marks.

However, some candidates were under the misapprehension that det **X** = $\frac{1}{-3a+2}$ and that

$$\det \mathbf{Y} = \frac{1}{6a-4} \ .$$

Answer: (a)(i) $\frac{1}{10}\begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix}$ (ii) $\frac{1}{5}\begin{pmatrix} 4 & -7 \\ 3 & 6 \end{pmatrix}$ (b) $\frac{2}{3}$

Question 5

- (i) The majority of candidates gained full marks for this part. Most candidates set out their work clearly, showing the steps necessary to obtain the required result.
- (ii) Most candidates made correct use of the result in **part (i)** in an attempt to solve the given equation. However, only a few candidates realised that $\cos \theta = 0$ was a valid part of the solution. Most candidates divided each side of the equation $\cot \theta \cos \theta = \frac{1}{3}\cos \theta$ by $\cos \theta$, thus 'losing' two solutions. Candidates should be reminded that when giving angles in terms of radians an accuracy of three significant figures is required, not one decimal place.

Answer: (ii)
$$\frac{\pi}{2}, \frac{3\pi}{2}, 1.25, 4.39$$

Question 6

- (a) Correct answers to each part of this question were given by the great majority of candidates.
- (b) More errors were made by candidates in this part, with many finding it difficult to work with combinations correctly. Parts (i) and (ii) were not as problematic as part (iii). Errors in parts (i) and (ii) tended to be from the use of permutations rather than combinations. In part (iii) most candidates realised that there were two cases to consider, but often used incorrect combinations or permutations. Candidates should consider the validity of their answers in the context of the question. It would be unlikely that the number of ways the twins can be in the same group is a number in the thousands, so candidates should try to identify the error in their logic.

Answer: (a)(i) 40 320 (ii) 720 (iii) 5040 (b)(i) 35 (ii) 1 (iii) 15



Question 7

- (a) For many candidates, this question was the first one that caused them problems. Many still managed to obtain a correct answer, often through a lot of unnecessary working. It was intended that candidates made use of the fact that v could be obtained by multiplying the given magnitude by a unit direction vector. Alternative methods were acceptable.
- (b) Most candidates dealt with the vector manipulation correctly and hence equated like vectors to obtain two equations which could be solved simultaneously. There were very few errors seen.

Answer: (i) $\begin{pmatrix} 48 \\ -90 \end{pmatrix}$ (ii) p = 2, q = 2 and p = 10, q = -38

Question 8

- (i) Most candidates realised that they had to integrate with respect to *x* in order to find the gradient as required. There were errors in the actual integration itself, with some candidates failing to make use of an arbitrary constant and then go on to find the value of this arbitrary constant.
- (ii) Most candidates realised that they had to integrate with respect to x in order to find the equation of the curve as required. There were errors in the actual integration itself, with some candidates failing to make use of an arbitrary constant and then go on to find the value of this arbitrary constant. Some candidates also mistakenly attempted to find the equation of a straight line using the gradient they found in **part (i)** and the given point.
- (iii) Although many correct solutions were seen, errors included finding the equation of the tangent rather than the normal and also using an initial gradient of 5 which some candidates erroneously obtained either from **part (ii)** or from an arithmetic error in **part (i)**. Other candidates failed to give the answer in the required form.

Answer: (i) $\frac{dy}{dx} = 3 - 2\cos 2x$ (ii) $y = 3x - \sin 2x - \frac{\pi}{4}$ (iii) y = -0.798x - 0.294

Question 9

- (i) Most candidates were able to use the area of the minor arc correctly in order to obtain the correct value of θ .
- (ii) Use of the cosine rule was expected in order to find the lengths of *AC* and *BC*. Errors in the calculation of the angle required to use the cosine rule meant that many candidates obtained an incorrect length. Some candidates also mistakenly assumed that the lengths of *AC* and *BC* were 20 cm. Perhaps some revision of the properties of circles is needed by some candidates. Most, if not all, candidates obtained the correct arc length *AB*.
- (iii) Careful consideration of the shape was necessary in order to formulate a plan to find the required area. Problem solving skills were tested here. The two obvious options were either (i) the area of the minor sector *OAB* and the area of each of the triangles *OCA* and *OCB* or (ii) the area of the triangle *ABC* and the area of the minor segment formed by the minor sector *OAB*.

Answer: (i) $\frac{2\pi}{5}$ (ii) 50.6 cm (iii) 122 cm^2



Question 10

This unstructured question was designed to test the problem-solving skills of candidates. They needed to form a plan of action by deciding what they actually needed to do to find the required area. Most candidates did have a correct plan and found the coordinates of the points *A* and *B* and hence the area of the triangle necessary to find the required area. Some chose to find the area of the trapezium which was not a great deal of use unless the area enclosed by the curve and the *y*-axis between the appropriate *y* values was used. Some candidates chose this approach with a mixed level of success. Some candidates opted to find the equation of the straight line *AB* and involve this in their integration required to find the shaded area. Often candidates made simple errors but were using a correct approach. It was important that candidates realised

that they should be working with exact values as they were required to give the answer in the form $\frac{p\sqrt{3}}{2}$

Too many resorted to the use of their calculator rather than manipulate the necessary surd forms.

Answer:
$$\frac{9\sqrt{3}}{20}$$

Question 11

- (i) Most candidates were able to make use of logarithms and find the value of *b* either by use of simultaneous equations or by using the gradient of the straight line graph. Some errors occurred when attempting to find the value of *A* when incorrect use of logarithms was more common. An exact value for *A* was acceptable but some candidates chose to give their answer to *A* to one decimal place rather than the required three significant figures.
- (ii) Most candidates were able to use a correct method in order to find the required value of *y*, with many correct solutions being seen.

Answer: (i) b = 2, A = 0.273 or $e^{-1.3}$ (ii) 14.9



ADDITIONAL MATHEMATICS

Paper 0606/22 Paper 22

Key messages

In order to do well in this examination, candidates need to give clear and logical answers to questions, with sufficient method being shown so that marks can be awarded. Candidates need to give attention to instructions in questions such as 'Without using a calculator', or 'showing all your working'. These instructions indicate that omitting method will result in significant loss of marks. Candidates who omit to show key method steps in their solution to a question through using a calculator, even when permitted, risk losing marks, should they make an error. Candidates should ensure that their answers are given to no less than the accuracy demanded in a question. When no particular accuracy is required in a question, candidates should ensure that they follow the instructions printed on the front page of the examination paper. Careful attention should be given to the accuracy required for angles in degrees, which varied from those in radians.

General comments

The majority of candidates were well prepared for this examination. They showed good knowledge of key concepts and many candidates were able to apply techniques successfully.

Some candidates used extra sheets to carry out 'rough calculations'. These calculations were usually then deleted and marked as rough work. This work was often poorly presented and difficult to read. Occasionally, marks can be awarded for rough work that has been deleted and not replaced. This can only be done if the work is readable. Candidates who use this approach also, occasionally, miscopy their own work. For these reasons, candidates would do better to write all of their working as part of their main solution to a question. Clear and well-presented solutions indicate clear and logical thinking. Candidates who presented a whole solution in this way usually scored highly. Other candidates need to understand that, when their work is difficult to understand, it is difficult for marks to be awarded. Also, often candidates misread their own writing when presentation is poor and accuracy is then lost.

Many candidates gave fully worked solutions, showing **all** their method clearly. Other candidates could improve by showing all the key steps of their method. This is even more important if they make an error. Showing clear and full method is essential if a question asks candidates to 'Show that...'. This instruction indicates that the answer has been given and that the marks will be awarded for showing how that answer is found. The need for this was highlighted in **Question 3**, **Question 6(b)(ii)** and **Question 11(iv)** in this examination. Showing clear and full method is also very important when the use of a calculator is not allowed, as was the case in **Question 5**.

When a question demands that candidates 'Explain why' something is valid or correct, it is important that the explanation is supported with a valid reason. Comments made without justification are often just restatements of the information given in the question. This was evident in **Question 6(b)(i)** and **Question 11(ii)** in this paper.

Candidates seemed to have sufficient time to attempt all questions within their capacity.



Comments on specific questions

Question 1

Almost all candidates gave a correct solution to this question. The majority separated the information into two correct linear equations and solved them. Some candidates formed a correct quadratic and solved that. Very few candidates gave incorrect solutions. One or two candidates thought that $x = -\frac{5}{3}$ was not a valid solution and discarded it. Clearly these candidates are confused about the meaning of the absolute value.

Answer: $x = 5, -\frac{5}{3}$

Question 2

- (i) This question was almost universally correct. A very small number of candidates calculated $12000 \times e^{-0.2}$ instead of $12000 \times e^{0}$.
- (ii) Again, this question was very well answered. Almost all candidates evaluated a correct calculation and gave a sufficiently accurate final answer. A few candidates prematurely approximated their values. These candidates would improve should they realise that this resulted in an inaccurate final answer and the loss of the accuracy mark for the question. Some candidates also need to be aware that exact answers, such as $-2\ln\frac{2}{3}$, are not appropriate when the question is in context, as in this case. A small number of candidates slightly misinterpreted what was required. These candidates found the time it took the value to decrease to $\frac{2}{3}$ rather than $\frac{2}{3}$ of the value when new. These candidates would likely improve if they read the question a little more carefully.

Answers: (i) 12000 (ii) 2.03 years

Question 3

- (i) A large proportion of the answers presented for this question were neat and correct. The most successful candidates chose either to multiply out the linear and cubic factors to show the quartic was obtained or used synthetic division. Candidates using more formal algebraic long division often made slips with powers or with signs. Some candidates would have improved if they had taken a little more care with presentation, as often this was the cause of any error made. A few candidates used the factor theorem to show that x = 1 was a root. This, on its own, was insufficient to demonstrate what they were being asked to show and therefore was not credited.
- (ii) In this part of the question candidates were instructed to show all their working. Many candidates did so and were awarded full marks. Other candidates would have improved if they had shown the working that supported their choice of a second linear factor. Stating '*By trial and improvement method*' and then writing down a factor is not sufficient. Clear evidence that the factor stated was indeed a factor needed to be seen. Many candidates were successful in doing this by using the factor theorem or using algebraic long division or synthetic division. Some candidates showed that (x-1) was a common factor of $x^3 x^2$ and -4x + 4. This resulted in a very quick and neat solution. A small number of candidates factorised correctly but then stated their final answer as the set of solutions to p(x) = 0, which was not required. A few other candidates would probably have corrected this omission if they had reread the question after they had written down their final answer.

Answer: (ii) $(x-1)^2(x+2)(x-2)$



Question 4

This was another very well-answered question. All candidates realised that equating the expressions for the line and curve then considering the discriminant of the resulting quadratic was a good method of solution. The majority of candidates chose the correct condition, $b^2 - 4ac < 0$, and applied it correctly. A small number of candidates needed to take more care, as sign errors were seen on occasion. Some candidates solved $b^2 - 4ac = 0$, $b^2 - 4ac > 0$ or $b^2 - 4ac \le 0$. These candidates may have avoided this type of error if they had a better understanding of the relationship between the discriminant and the given graphs.

Answer:
$$k < -\frac{1}{2}$$

Question 5

Many clear and accurate solutions were seen to this guestion. Most candidates gave sufficient evidence that they had solved the problem without using their calculator. Candidates who showed $\sqrt{20} = \sqrt{4 \times 5}$ and then wrote this as $2\sqrt{5}$ almost always scored full marks. Many candidates were given the benefit of the doubt where $2\sqrt{5}$ was used as $\sqrt{20}$. This was allowed as long as there was enough evidence later in the question to conclude that a calculator had not been used. A small number of candidates arrived at the correct answer without sufficient evidence of their method. These candidates may have gained more marks if they had shown full working, as required.

Answer: $1+\sqrt{5}$

Question 6

(a) (i) A good number of candidates correctly manipulated the terms of the product and found the correct expression. Candidates whose first step was of the form $(-8x^9)^{\frac{1}{3}}(x^{-3})^{\frac{1}{6}}$ were the most successful. Candidates who were not fully correct would have done better if they had taken a little more care with their first step. Often, candidates who made errors dealt with the -8 incorrectly. In many cases

it remained as -8 following a first step of $-8x^{\frac{9}{3}}$ or it was cubed to become -512.

- (ii) Most candidates seemed to understand what was required here and equated their answer from part (a)(i) with -6250 then solved. It was rare for any candidate to check their answer by substituting their 25 into $(\sqrt[3]{-8x^9})(\sqrt[6]{x^{-3}})$ to make sure that its value was -6250. Those candidates whose answer to part (a)(i) was incorrect may have been alerted to a possible error if they had carried out this simple check.
- Candidates usually identified the second term in the expression for y as being the term of interest (b)(i) in their explanation. Many gave clear explanations, either using words or solving a simple inequality. Candidates considering 4x - 3 > 0 were usually successful. Candidates considering values of x that were not valid were much less successful. The majority of candidates arguing from this standpoint either only considered x = 0.75 or x < 0.75. Both cases were needed to justify x > 0.75. Some explanations were not sufficiently clear. Stating, for example, that y does not exist because a log cannot be negative is not acceptable as the reason given is too vague.
 - As the answer has been given to candidates, complete and clear method needed to be shown. A (ii) good proportion of the candidates did earn full marks here. Those who did not needed to justify their calculations. For example, it was reasonably common for these candidates to earn the mark for dealing with the power in the second term. Fewer candidates earned the mark for demonstrating why the log_a a terms disappeared. Some candidates simply removed all the as from their working without any justification at all. Some candidates were not careful in their writing down of the given answer. These candidates needed to check the expression given once they had finished the question.



(iii) This question was very well answered. Most candidates understood the connection with the previous part of the question and used the expression correctly. A high proportion of candidates also correctly discarded the solution x = 0. A very few candidates did not make the connection with the previous part of the question. These candidates generally tried to rewrite the equation as an exponential relationship but made no progress with the solution.

Answers: (a)(i) $-2x^{\frac{5}{2}}$ (ii) 25 (b)(iii) $\frac{3}{2}$

Question 7

- (a) Candidates found this question more challenging. Success was generally dependent on candidates drawing a reasonable diagram and using it correctly. Various correct diagrams were possible. Candidates who drew an obtuse-angled vector triangle representing the sum of the two vectors almost always used it correctly to find the magnitude and direction required. This was the most straightforward method of solution. Candidates who drew a diagram with the velocities acting at a point often misused it rarely did they use it to find the horizontal and vertical components of the resultant vector. Mostly, a diagram of this form resulted in a cosine rule attempt with an angle of 60° rather than 120°. At least one attempt at a scale drawing was seen. Candidates were asked to calculate the values and scale drawings are not accepted in this case. Some candidates did not use their angle correctly to find the bearing. Knowledge of bearings is assumed at this level. Some candidates forgot to find the direction/bearing once they had found the magnitude. These candidates needed to read the question more carefully or reread the question once they had finished it to ensure they had completed it.
- (b) A few candidates gave fully correct answers, remembering to include the direction in their answer. A reasonable number of candidates found the correct speed without stating the direction. Many candidates simply subtracted the given speeds, not taking note of the opposite direction of travel.

Answers: (a) 13.2 ms^{-1} on a bearing of 220.9° (b) 94[km/h], west

Question 8

- (i) This question was very well answered by almost all candidates. Generally, the errors made were few and these tended to be sign or arithmetic slips rather than fundamental method errors. A few candidates unnecessarily found the coordinates of the midpoint of the line *AB* and used that point to form their equation. This was, fortunately, usually done without error.
- (ii) Again, candidates found this straightforward. Almost all candidates understood the need to rearrange the equation of the given line in order to find its gradient. The perpendicular gradient was generally correctly found and a form of the correct equation stated. A very small number of candidates either did not find the gradient of the given line correctly or tried to find the equation of

the line perpendicular to L. One or two candidates gave equation of line = $-\frac{3}{2}x + 25$ as their

answer. This was not acceptable and it is expected that candidates will write their answers in a correct Cartesian form.

Answers: (i) (18, -2) (ii) $y = -\frac{3}{2}x + 25$

Question 9

- (a) Candidates clearly knew how to integrate expressions of this type and most did so correctly. Very few candidates altered the power and most correctly divided by 2. A good proportion of candidates also gave a constant of integration as part of their solution, showing good understanding of what was needed for a complete solution.
- (b) (i) Very many good and fully correct solutions were seen here. Most candidates applied a correct quotient rule. Although some correct product rule attempts were also seen, this was slightly more likely to have an error.



(ii) Most candidates demonstrated that they understood how the previous part of the question enabled

them to find the given integral. Those who started with $\frac{x}{\ln x} + \int \frac{1}{x^2} dx = \int \left(\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right) dx + \int \frac{1}{x^2} dx$

were most successful. A few candidates incorrectly subtracted $\int \frac{1}{x^2} dx$ and this usually resulted

from writing $\frac{x}{\ln x} = \int \left(\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right) dx$ and then adding $\int \frac{1}{x^2} dx$ to the right hand side of this

equation only. A good number of candidates correctly found $\int \frac{1}{x^2} dx$. Those who did not may have done so if they had realised the need to rewrite the expression as a power of *x*.

Answers: (a)
$$\frac{e^{2x+1}}{2} + c$$
 (b)(i) $\frac{dy}{dx} = \frac{\ln x - 1}{(\ln x)^2}$ (ii) $\frac{x}{\ln x} - \frac{1}{x} + c$

Question 10

(i) Most candidates scored at least the first two marks for successfully rewriting the equation and taking the inverse tangent of $\frac{4}{3}$. Very few candidates made errors with these initial steps. One or

two candidates either used an incorrect major trigonometric ratio or stated $\frac{1}{\tan}(2x-10) = \frac{3}{4}$ and

did not recover a correct form from this error. Some candidates were penalised for making premature approximations when finding the double angles which resulted in their final set of angles being slightly inaccurate. Other candidates gave only two angles rather than the four required. This was either because they considered double angles between 0° and 360° only because they did not include the reflex double angle 233.1. Many good candidates avoided these errors. They achieved this by writing down that $0 \le x \le 360$ means that $-10 \le 2x - 10 \le 710$ and/or by drawing a quadrant (CAST) diagram or a simple tangent curve.

(ii) Candidates answered this question very well. Most of them understood the need to write the equation in terms of cosine only and correctly solved the resulting quadratic equation. Some candidates used a base angle of 0° when finding the inverse cosine of –1. Although this was usually well done, a few candidates using this approach incorrectly included 0° and, occasionally, 360° as an answer. Very few candidates showed any evidence of checking their angles in the original equation. A check such as this would help candidates who have erroneous angles in their solutions to discard them.

Answers: (i) 31.6°, 121.6°, 211.6°, 301.6° (ii) 60°, 300°, 180°

Question 11

- (i) A very high proportion of candidates gave a correct answer to this part.
- (ii) Many candidates scored at least one of the two marks available and a reasonable number scored both marks. When asked to 'Explain why' something is valid, the explanation given should always be fully justified. The most straightforward explanation here was to calculate g(1) as 0 and to observe that 0 is not in the domain of f. Some candidates realised that the domain of f was the issue but then stated that 1 was not in the domain of f. Whilst this was correct, it did not explain why fg(1) did not exist. Other candidates gave partially correct or partially complete explanations. For example, a few candidates stated that f(0) was not possible without justifying why it was not possible.



- (iii) A good number of fully correct solutions were seen. Almost all candidates composed the function in the correct way and squared $\frac{x^2-2}{x}$, or its simplified equivalent, correctly. Candidates not earning all three marks commonly either calculated -4 1 as -3 or made the arithmetic slip of doubling rather than halving in the final simplification of the expression.
- (iv) A very good number of correct answers were given. A small number of candidates gave an answer of $gf \ge 2$, which was not allowed. Other candidates gave the domain of g rather than f as their answer. Some candidates appeared to be trying to use the expression they had found in **part (iii)** rather than knowledge that the domain of gf is the same as the domain of f. These candidates usually gave $x \ne 0$ as their answer.
- (v) Candidates were asked to show the inverse of f was of a particular form. This question proved to be a good discriminator with usually only the very best candidates scoring three or four marks. Candidates giving the best solutions started with the function f and rearranged to find a correct quadratic and then applied the quadratic formula. The discarding of the negative square root needed to be justified. This was rarely seen. Most candidates simply ignored the negative case or commented that it should be ignored without saying why. Many candidates verified that the inverse of the given function was f or were clearly working using the given inverse function. These candidates were partially credited for their solutions. When candidates are asked to show that a result is true, they should not assume it is true and use it as part of their solution. Some candidates made very little progress with their solution and it may be that these candidates need more practice in finding inverses for functions of this type.

Answers: (i)
$$g \ge -\frac{1}{2}$$
 (iii) $\frac{x^2}{2} - \frac{5}{2} + \frac{2}{x^2}$ (iv) $x \ge 2$

Question 12

(i) Many candidates saw the simplest solution – to find the length of the side of the rectangle using the information given and then subtract the area of the triangle from the area of the rectangle. Most of these candidates did this successfully. Some candidates made sign errors when simplifying to the given form. Some candidates were misled by the + in the numerator of the fraction in the brackets of the given expression and attempted to add the areas they should have been subtracting. These candidates would have done better if they had concentrated on the geometry and only checked their answer at the end, rather than trying to work to the answer. A small number of candidates separated the area into a small rectangle and two right-angled triangles. This was slightly more involved. A few of the candidates were successful, as they added valid areas to find the given expression. A small number of candidates would have presented better solutions if they had marked the dimensions they were using on the diagram they were given. Candidates who did not

annotate their diagrams often made errors such as using $\frac{x}{2}$ rather than x as the base of the

equilateral triangle that had been cut from the rectangle.

(ii) This proved to be a good final question for many candidates with a high proportion scoring well. A few candidates left the value of *x* in exact form rather than giving it to two significant figures, as required. Other candidates forgot to find the maximum area. Once again, rereading the question to check they had answered it completely may have helped these candidates. No fundamental method errors were seen.

Answer: (ii) x = 2.6, A = 13

